# Comparative Analysis of the Monte-Carlo Method and the Confidence Ellipsoids for Solving Task of REC Reliability Evaluation

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**Abstract.** *The problem of evaluating functional reliability REC using the Monte Carlo method and confidence ellipsoids. Was made an analyze these methods in terms of computational complexity, basing on which, using the programming language C #, was developed program for compare the efficacy of two methods. In the particular example shows that the same probability of functional fitness REC, Monte Carlo order of magnitude more costly in terms of time complexity than the method of confidence ellipsoids.*

## Keywords

Functional suitability, Monte-Carlo method,
confidence ellipsoid method.

## 1 Introduction

The concept of "functional reliability" or "reliability" of the device, is used long time in engineering practice. Any technical device always made on the basis of its long-term use for practical purposes. In this work will be considered the type of device in which reliability plays a significant role, as the particular system (the set of elements) that interact in the course of a given function. This reliability is the property of the system (object) stored in space and time within the prescribed limits values of all the parameters that characterize the ability to perform certain functions in the set modes and operating conditions [1].

Existing approaches to evaluating functional reliability [2,3] are based on the calculation of the reliability based on the calculation of multidimensional integrals which dependent on the system characteristics number. Among them, there are approximate methods, including Monte Carlo, which has both advantages and disadvantages. Also, currently, the evaluating of system reliability index can be obtained by using trusting ellipsoids [4].

Under these conditions, comparison of the two methods mentioned above on the example of reliability assessment REC based on the analysis of random deviations from the nominal parameters of REC becomes the actual problem.

## 2 Stating the task

### 2.1 Monte Carlo method

The emergence of simulation methods (Monte Carlo) in various areas of applied mathematics is usually associated with the need resolving of new practical tasks. Monte Carlo method is a numerical method for solving mathematical problems (systems of algebraic, differential, integral equations) and direct simulation (physical, chemical, biological,

economic and social processes) by using obtaining and conversion of random number. In the analysis of functional reliability elements of the systems, most frequency are used following laws of distribution of random variables: exponential distribution, normal distribution, the distribution Veybula etc. [5].

The general scheme of the Monte Carlo method is based on the central limit theorem of probability theory, which asserts that a random variable $Y = \sum_{i=1}^{N} X_i$ is the sum of a large number $N$ of arbitrary random variables $X_i$ with the same expectation $m$ and variance $\sigma^2$, are always distributed by normal law with mathematic expectation $N \cdot m$ and variance $N \cdot \sigma^2$. Normal distribution characterized by a probability density:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-\frac{(x-m)^2}{2\sigma^2}} \tag{1}$$

where $m$ - mathematical expectation of value $X$, and $\sigma^2$ - dispersion of the $X$ value [6].

In the basis of the Monte Carlo is a random number generator - an algorithm that generates a sequence whose elements are almost independent of each other and match the law. In practice, in most cases, are used programmed methods of generation.

The main advantages of the Monte Carlo include the following:
- taking into account the full functioning of the systems;
- simple schemes of computing algorithm;
- possibility of simulation of random variables with given distribution laws and etc.

However, beside the above-mentioned advantages, Monte Carlo method has some disadvantages, the most significant of which are: high complexity, partial nature of resolving, and also increasing the computational complexity of the algorithm with increasing number of iterations.

To reduce the impact or even entirely avoid the above mentioned drawbacks on reliability problem solution evaluation functional reliability device makes it possible to combine methods of interval analysis method and confidence ellipsoids.

## 2.2 Method of constructing tolerance region based on the analysis of interval data

Consider the REC as a black box as shown in figure 1.

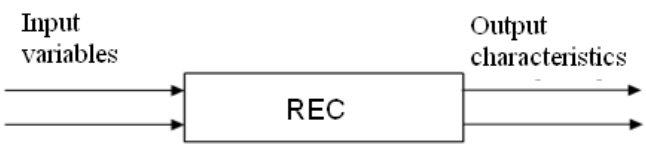


**Fig. 1**. REC model as a "black box".

Nominal values of REC elements could be represented as input variables, including resistance, capacitance, inductance, etc.. Baseline characteristics could be, for example of damping factor at a certain frequency, amplitude-frequency characteristic REC of the certain voltage across the element and so on.

Traditionally, when setting tolerances radioelements REC the limits on the output characteristics are set as [7]:

$$y_i \in [y_i^-, y_i^+], i = 1,..., N \tag{2}$$

and established the relationship between the parameter values and the corresponding output characteristics

$$y_i = g_i(\vec{b}), i = 1,..., N. \tag{3}$$

Given that the in general case characteristics of the REC is nonlinear, it is reasonable to use linearization of REC characteristics and move to a linear system of inequalities of the form:

$$\delta\vec{Y}^- \le S \cdot \delta\vec{b} \le \delta\vec{Y}^+, \tag{4}$$

where $\delta\vec{Y}^- = \{\delta y_i^-, i = 1,..., N\}, \delta\vec{Y}^+ = \{\delta y_i^+, i = 1,..., N\}$ - vectors which composed of the upper and lower limits of the intervals deviation of output characteristics from the nominal; $S = \{S_{ij}, i = \overline{1, N}, j = \overline{1, m}\}$ - known matrix values derived functions $g_i(\vec{b})$ at the point $\vec{b}_0$; $\delta\vec{b} = (\delta b_1,..., \delta b_m)^T$ - vectors relative REC parameters deviations.

By the condition of the system compatibility (4) region of it's solutions $\Omega$ will be options [8]:

$$\Omega = \left\{\delta\vec{b} \in R^m \middle| \delta\vec{y}^- \le S^T \cdot \delta\vec{b} \le \delta\vec{y}^+\right\} \tag{5}$$

where $\delta\vec{y}^- = \left\{y_i^-, i = 1,..., N\right\}$, $\delta\vec{y}^+ = \left\{y_i^+, i = 1,..., N\right\}$ – vectors of upper and lower limits of the intervals for the deviation characteristics of REC from the nominal.

Given the complexity of the tolerance region descriptions, the complexity of which will be increased in several times with the increasing of the radio elements number, it is advisable to use various approximate methods of evaluation of this area. One method of evaluation is a method of approximation tolerance region using confidence ellipsoids.

## 2.3 Method of confidence ellipsoids

Ellipsoid in the space of realizations of random vector is characterized by concentration of its probability distribution near some vector which is established, in terms of moments of the 2nd order. Let the random vector $\delta\vec{b}$, which takes the value $\delta\vec{b} = (\delta b_1,..., \delta b_n)^T$ in $n$- dimensional Euclidean region $R^n$, has non-degenerate covariance matrix $D^{-1}(\delta\vec{b})$. In this case, for any fixed vector $\delta\vec{\bar{b}}$, $\delta\vec{\bar{b}} \in R^n$ in the region realizations $R^n$, you can define an ellipsoid [9]:

$$Q(\alpha, m) = \{\delta\vec{b} \in R^n \middle| (\delta\vec{b} - \delta\vec{\bar{b}})^T \cdot D(\delta\vec{b}) \cdot (\delta\vec{b} - \delta\vec{\bar{b}}) \le \chi^2(\alpha, m)\} \tag{6}$$

where $\chi^2(\alpha, m)$ - quantile $\chi^2$ -distribution.

Rather often the intervals of REC output characteristics are symmetric with respect to their nominal values, hence the centers of symmetry tolerance region and ellipsoid coincide and are located at zero point, in order to inscribed ellipsoid touches with all facets of tolerance region. As the conclusion, the value of $\delta\vec{b}$ are constant and only covariance matrix ellipsoid influence on the capacity of the ellipsoid.

In the work [4] was considered the method of evaluating the reliability of REC in the case of known covariance matrix technological or operational deviations of parameter vector from the nominal values. It was established conditions of belonging confidence region to tolerance:

$$\chi^2(\alpha, m) = 1 \Big/ \max_{i=1,..,m} \{\Lambda'_{ii}\} \tag{7}$$

from which the reliability of REC is defined as confidence probability $\alpha$ for calculating values for quantile $\chi^2(\alpha, m)$. Let move to comparative analysis of the computational complexity of the above described methods.

## 2.4 An example application

For the comparing of the two methods described above let consider the functional suitability of the low-pass filter with parallel connected elements. Nominal values of resistance and capacitance are as follows: $R_0 = 0{,}5kOhm$ та $C_0 = 0{,}5\mu F$. Nominal values of coefficient transfer module are considered at two frequencies: $f_1 = 1kHz$, $f_2 = 2kHz$. Requirements for the functional suitability of the filter (Fig. 2) will be set at 15% probability of deviation module of the at those frequencies.

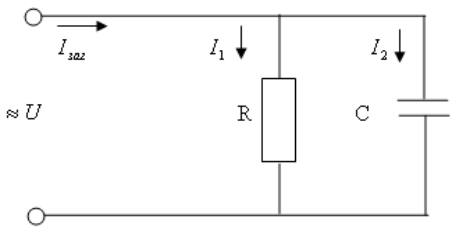


**Fig.2.** Scheme of the lowpass filter.

Frequency response for the nominal values of the parameters element lowpass filter will look like:

$$| K |= \frac{1+R^2w^2C^2}{R^2+w^2C^2} \Rightarrow K_{oi} = \sqrt{\frac{1+R^2w^2C^2}{R^2+w^2C^2}} \tag{8}$$

Due to non linearity characteristics of the filter, hold linearization using parameters of amplitude-frequency characteristics elements in the vicinity of the nominal values and determine the sensitivity characteristics of the filter to modify the elements.

$$\partial K_i^- \le \sum_{j=1}^{m} S_{ij} \cdot \partial b_j \le \partial K_i^+, i = 1,.., N, \tag{9}$$

where $\partial \vec{b}_j = (\partial R, \partial C)$, $\partial K_i^- = K_i^- - K_{0i}$, $\partial K_i^+ = K_i^+ - K_{0i}$.

Using the C # programming language it was developed algorithmic implementation of the above described methods, in order to conduct a comparative analysis of the computational complexity of each. As a result of the method of confidence ellipsoid formula (7) is calculated quantile values: $\chi^2(\alpha, m) = 4{,}84$, and using tables $\chi^2$ - distribution, defined probability of functional reliability REC: $P = 94\%$. For achieving this same probability of 94% in the Monte Carlo was necessary to generate 500 thousand random numbers. Comparison of the results of both methods is illustrated graphically in figure 3.

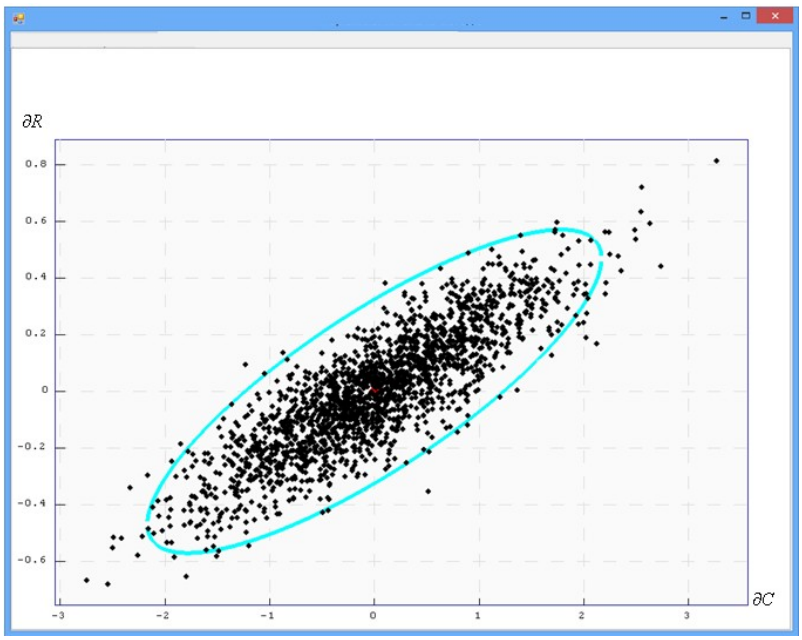


**Fig. 3.** Geographical comparison of two methods.

The main advantage of the confidence ellipsoids method is low time complexity. Specifically, as shown in Figure 4, to achieve the same precision techniques in the implementation of the method of confidence ellipsoids were spent approximately 0.6 seconds, which is in 22 times less than the realization of the Monte Carlo.

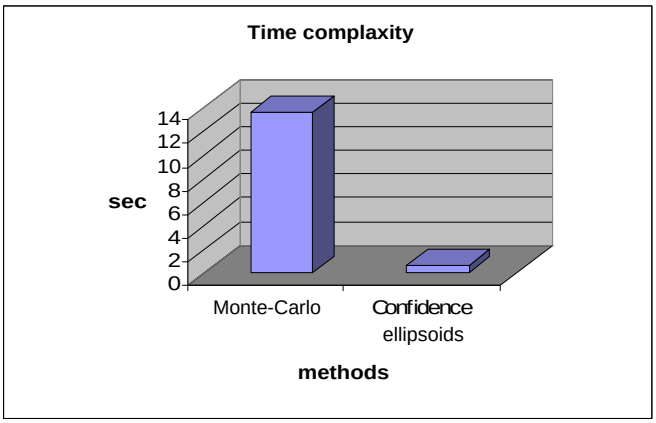


**Fig. 4.** The time complexity of the methods.

## 3 Conclusion

It was analyzed methods of evaluating of the functional reliability devices, including the method of simulation Monte Carlo method and interval analysis - the method of confidence ellipsoids. Was set a number of advantages Monte Carlo method and was presented its main disadvantages, which can be quite well avoided by using of the method specified confidence ellipsoid.

Performed a comparative analysis of the time complexity of both methods and a specific example shows that for the same accuracy of the method, the time complexity of the Monte Carlo method dominates over the time complexity of confidence ellipsoids in 22 times.

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